Detection of conductivity fluctuations in a turbulent flow field

By C. H. GIBSON† AND W. H. SCHWARZ
Department of Chemical Engineering, Stanford University, Stanford, California

(Received 17 January 1963)

A technique has been developed to detect very small amplitude fluctuations of concentration or temperature in aqueous electrolytic solutions. A platinized single-electrode conductivity probe is made one arm of an A.C. Wheatstone bridge, whose unbalance is then proportional to the incremental change of conductivity at the probe tip. A discussion of the sensitivity and spatial resolution of the probe and circuitry is given. This technique should have useful applications in oceanographic measurements of salinity or temperature, measurements of statistical parameters useful in the description of diffusion-controlled reactions in liquid-phase chemical reactors, and also the pure mixing of ionic materials and temperature.

1. Introduction

The measurement of the statistical parameters which describe the mixing of scalar fluid properties such as temperature or electrolyte concentration by a turbulent flow field is important in the understanding of practical systems like mixers and chemical reactors as well as in fundamental investigations of the turbulent mixing processes. The information obtained from 'point' values of the scalar field are valuable for comparison with existing theories of turbulent mixing as well as providing empirical data to calculate the scattering of acoustic and electromagnetic waves by a random scalar field.

Since the conductivity of electrolyte solutions depends on both temperature and concentration, the local variation of these properties in a body of electrolyte can be measured with a conductivity probe with cell volume small with respect to the length scale of the temperature or concentration fluctuations. A suitable electrical circuit may then be used to convert the fluctuations of the impedance of the conductivity cell to a proportional electrical signal which may be further analyzed by conventional techniques. For example, the electrical power and power spectrum of the signal may be determined and converted to the corresponding physical descriptions of the structure of the conductivity field.

In this paper, a conductivity probe and electrical circuit will be described which have been used to measure the turbulent mixing of both temperature and salt concentration fields behind a grid in a water tunnel. The results of that work are given in Gibson & Schwarz (1963).

---

† Present address: Chemical Technology Department, Osmania University, Hyderabad (A.P.), India.

2. Previous related work

The use of platinized-electrode conductivity cells in A.C. circuits to determine the resistivity of electrolyte solutions originated with Kohlrausch in 1875 (Glasstone 1942, p. 32). Grinnell Jones and associates (1928 *et al.*) carefully investigated the effects of many of the variables and refined the techniques of precise measurements of the conductivity of uniform solutions.

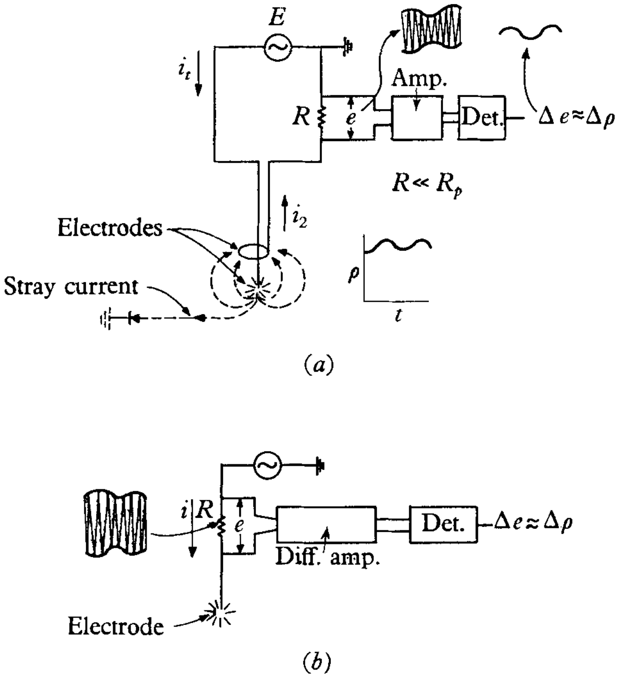
Prausnitz & Wilhelm (1956) and Lamb, Manning & Wilhelm (1960) developed conductivity techniques for measurements in non-uniform electrolytic solutions. Prausnitz & Wilhelm (1956) constructed a conductivity cell with two small electrodes close to each other so that the resistivity of the cell would be determined primarily by the resistivity of the solution between the electrodes. They used this cell to measure salt concentration fluctuations in a packed bed. Subsequent measurements have been made in a fluidized bed by Cairns & Prausnitz (1960*a*,*b*).

Lamb *et al.* (1960) and Manning (1959) also used a two-electrode probe, but reduced the cell volume of the probe considerably by making one electrode much larger than the other so that the resistance of the fluid between the electrodes would be dominated by the resistivity of the solution very close to the smallest electrode. The larger electrode collected the current passed by the small electrode, generating an amplitude-modulated carrier signal across a small metering resistor between the large electrode and ground which could be amplified and demodulated to give an electrical signal proportional to the resistivity fluctuations near the small electrode. One disadvantage of this arrangement was that some of the current might be lost to stray grounds in the system. Another disadvantage is that if the fluctuations of conductivity in the solution are small, the percentage modulation of the carrier signal will also be small and such factors as noise and oscillator ripple may be as large as the conductivity fluctuation signal hence restricting the circuit's use to large conductivity fluctuations. The operation of this circuit is illustrated in figure 1 (*a*).

3. Single-electrode conductivity probe

The influence of the larger electrode on the cell resistance of a two-electrode probe may be entirely eliminated by allowing its area and separation from the smaller electrode to become large. For example, a grounded metal wall of the experimental system could be used as the second electrode of the cell, hence a single electrode conductivity probe might be used. The possibility of using single-electrode conductivity probes was also suggested by Dr G. K. Batchelor and Dr H. K. Moffatt in 1961, and by Dr N. Keeler and Dr K. M. Kiser in 1962. Keeler has used a single electrode probe in the circuit shown in figure 1 (*b*), where the metering resistor is between the small electrode and the oscillator so that a differential amplifier must be used to amplify the carrier signal. Although this circuit eliminates the influence of stray grounding, it has the same inherent disadvantage as the circuit of figure 1 (*a*) in that it measures the total resistance of the cell, rather than just the fluctuating component which is really the quantity of interest. In addition, the differential amplifier must have a high rejection ratio or a portion of the common mode will have to be amplified along with the carrier.

A schematic diagram of a single-electrode conductivity probe is shown in figure 2. The shielded lead from the bridge circuit is soldered to a platinum wire which passes through a $\frac{1}{4}$ in. diameter glass tube to the cast epoxy tip containing the small electrode wire. Both ends of the 8 in. long tube are sealed with epoxy. The tip of the probe is carefully shaped with jeweller's files and needle points under a microscope to obtain the optimum sharpness and a clean surface for platinizing before each usage. The electrode diameter of the finest probe used was 0·0004 in. Both the electrode size and the hydrodynamic disturbance are critical in the final shaping of the probe tip since one or the other will control the actual spatial resolution of the probe. To minimize the surface polarization and 'noise' at the probe surface, approximately 10 C/cm^2 of platinum black are deposited on the exposed electrode surface from a standard platinizing solution while the probe tip is being observed through a microscope.



(*a*)

(*b*)

FIGURE 1. Circuitry of amplitude modulated conductivity probes.

Spatial resolution

An estimate of the spatial resolution or effective cell volume seen by the probe may be made in the following way. If a spherical electrode with radius r_0 is immersed in a static body of solution with uniform resistivity $\rho(\Omega\,\mathrm{cm})$, the resistance between the body and a spherical shell of radius B surrounding the body is given by

$$R_B = \frac{\rho}{4\pi}\left(\frac{1}{r_0} - \frac{1}{B}\right). \tag{1}$$

Then, as $B \to \infty$,

$$R_\infty = \rho/4\pi r_0 = K_c \rho, \tag{2}$$

where K_c is the cell constant of the probe. An effective cell volume may be defined as a sphere whose radius gives a resistance equal to $0{\cdot}9R_\infty$. This is calculated to be ten times the spherical electrode radius and may be used as a rough estimate of the length scale of the scalar field resolved by the probe. The above analysis is crude for the following reasons. First, the electrode is not spherical, but resembles a hemispherical cap. Further the radius r_0 is dependent on the amount of platinization of the tip and hence is not readily estimated, and also the solution is neither static nor homogeneous. It seems natural however to define an effective probe diameter as

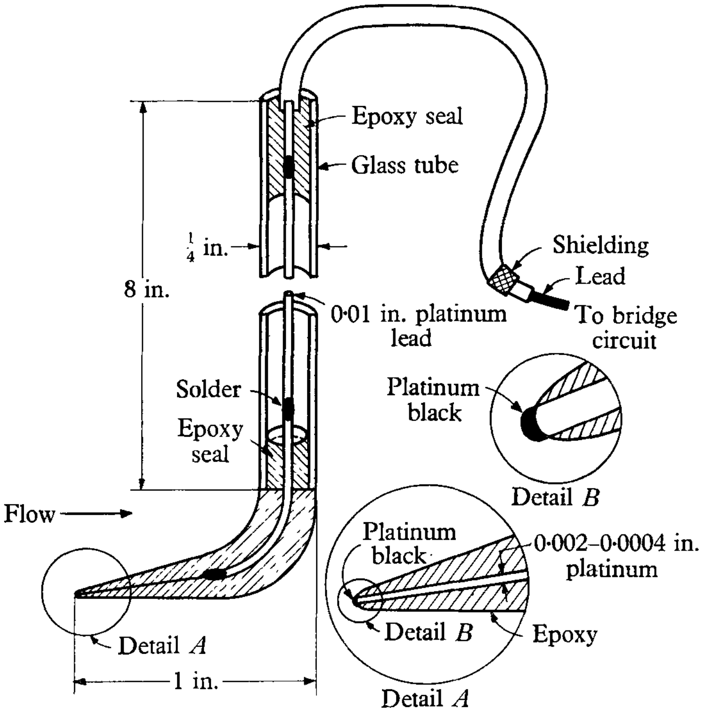


FIGURE 2. Single conductivity probe.

$$D_e = 1/2\pi K_c, \tag{3}$$

where the cell constant K_c is measured *in situ*.

To estimate the effect of spatial inhomogeneities, the following model may be used. Consider a spherical electrode of radius r_0 maintained at constant voltage E_0 in a large volume of fluid which has a spherically symmetrical variation of conductivity given by

$$\rho(r,t) = \rho_0 + A \sin \omega(r - r_0 + Ut), \tag{4}$$

and A, ρ_0, ω, r_0, and U are constants. A sketch of this resistivity variation is given in figure 3. The resistance between the electrode and ground is given by

$$R_p(t) = \frac{1}{4\pi}\int_{r_0}^{\infty}\frac{\rho(r,t)}{r^2}\,dr. \tag{5}$$

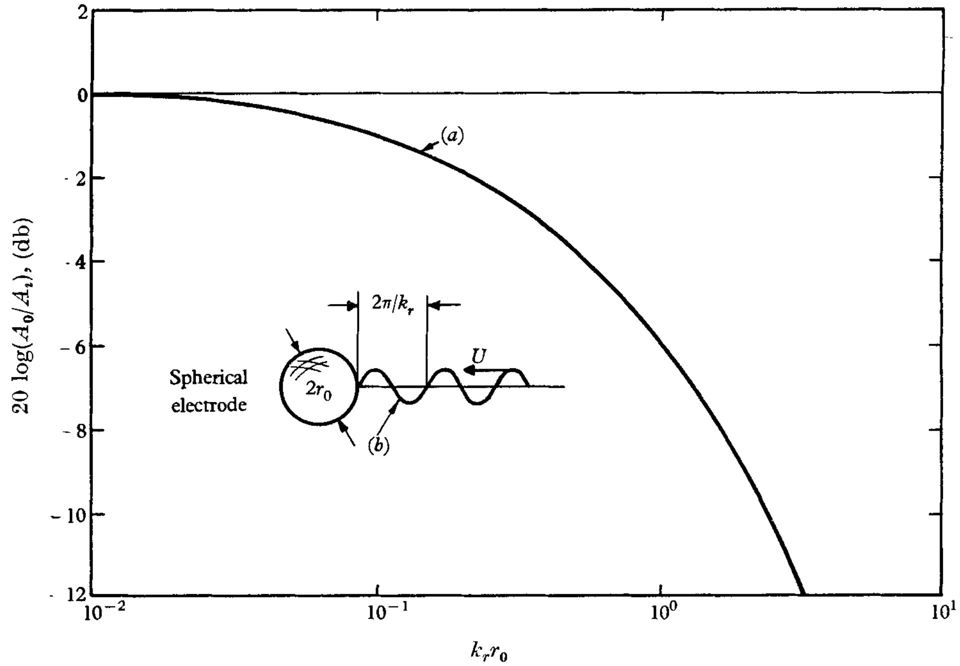


FIGURE 3. Spatial resolution model.
(a) $A_0/A_i = \{(\sin k_r r_0 - k_r r_0 \mathrm{Ci} k_r r_0)^2 + (\cos k_r r_0 + k_r r_0 \mathrm{Si} k_r r_0)^2\}^{\frac{1}{2}}$;
(b) $\rho(t,r) = \bar{\rho} + A_i \sin k_r(r - r_0 + Ut)$.

Substituting (4) into (5) and integrating, we obtain

$$R_p(t) = \frac{1}{4\pi}\left[\frac{\rho_0}{r_0} + A(k_1^2+k_2^2)^{\frac{1}{2}}\sin\left\{\omega(r_0 - Ut) + \sin^{-1}\frac{k_1}{(k_1^2+k_2^2)^{\frac{1}{2}}}\right\}\right], \tag{6}$$

where
$$k_1 = \omega[(\sin\omega r_0/\omega r_0) - \mathrm{Ci}\,(\omega r_0)], \tag{7}$$

and
$$k_2 = \omega[(\cos\omega r_0/\omega r_0) - \mathrm{Si}\,(\omega r_0)]. \tag{8}$$

Si and Ci are the sine and cosine integral functions, respectively. If the probe had a resistance equal to that of an ideal probe immersed in a uniform solution with the conductivity of the solution at the surface of the probe, namely

$$\rho(t) = \rho(r_0, t) = \rho_0 + A\sin\omega Ut, \tag{9}$$

the ideal probe resistance R_{pi} would be

$$R_{pi} = (\rho_0 + A\sin\omega Ut)/4\pi r_0. \tag{10}$$

Now, the instantaneous probe resistance may be separated into a mean part and a fluctuating part

$$R_p = \bar{R}_p + R_0(t); \quad R_{pi} = \bar{R}_p + R_i(t). \tag{11}$$

Then
$$R_i(t) = (A/4\pi r_0)\sin\omega Ut, \tag{12}$$

and from (6)

$$R_0(t) = \frac{A(k_1^2+k_2^2)^{\frac{1}{2}}}{4\pi}\sin\left[\omega(r_0-Ut)+\sin^{-1}\frac{k_1}{(k_1^2+k_2^2)^{\frac{1}{2}}}\right]. \tag{13}$$

The ratio of the amplitude of the fluctuations of resistance of the real electrode to that of the ideal electrode is obtained from (12) and (13) and is given by

$$\frac{A_0}{A_i} = \{[\sin\omega r_0 - \omega r_0\,\mathrm{Ci}\,(\omega r_0)]^2 + [\cos\omega r_0 + \omega r_0\,\mathrm{Si}\,(\omega r_0)]^2\}^{\frac{1}{2}}. \tag{14}$$

This function is shown figure 3. For a conductivity field given by (4) with a wavelength of $10r_0$, the power density would be down about 50% and for $100r_0$, about 10%. A wavelength of 5μ corresponds to a wave number of $1{\cdot}26\times10^4\,\mathrm{cm}^{-1}$. Therefore probes which are constructed from a platinum wire of 0·0004 in. (10μ) would have the output signal attenuated by about 10% at a wave number of $126\,\mathrm{cm}^{-1}$. Therefore, the very high wave-number regions of the scalar spectra must be corrected for this effect. This problem is similar to that of correcting hot-wire data for the finite length of the hot wire.

Some experiments using this probe (Gibson & Schwarz 1963) indicate that the attenuation of the signal calculated from this model may be slightly lower than actually measured, and is probably due to hydrodynamic effects at the probe tip. Although the electrode is quite small, the probe support will influence the structure of the turbulence in an unknown way. This deleterious effect may be somewhat diminished by a more suitable design of the probe.

4. Circuit

Figure 4 shows a schematic diagram of the circuitry. The conductivity probe is used as one arm of an A.C. Wheatstone bridge circuit which generates an electrical signal proportional to incremental variations in the probe impedance. These are proportional to fluctuations in the electrical conductivity of the solution at the probe tip, which in turn are proportional to the fluctuations in either concentration or temperature of the salt water. Two arms of the A.C. bridge are identical impedances and the bridge is balanced by adjusting a calibrated variable impedance until a null point is reached, a condition only possible when the variable impedance matches that of the probe. Fluctuations in the probe impedance due to variations in solution conductivity will cause a carrier-suppressed, amplitude-modulated unbalance voltage to appear across the bridge which may be demodulated to give the desired signal. The bridge circuit used was a commercial Tektronix Type Q Transducer and Strain Gage Oscilloscope Plug-In Unit which was modified slightly to reduce the level of the bridge activation voltage of 0·6 V R.M.S. per arm. The reduction was necessary both to reduce probe polarization and to prevent heating of the solution at the probe tip. The bridge circuit of the Q-unit is driven by a 25 kc/s carrier and the circuitry is arranged so that the demodulated output signal is down about 3 db at 6 kc/s. It may be possible to alter this device to improve the frequency response, although for many purposes it is quite adequate.

The bridge output voltage $e(t)$ was then analysed for mean-square and mean-power spectrum using a Ballantine 320 true-R.M.S. meter and a Hewlett–

Packard model 302A wave analyser (fixed band width 6·1 hertz). Since the readings of both of these instruments are generally much too erratic for accurate visual estimates of the mean, an analogue circuit was used to perform the necessary averaging. From the R.M.S. meter, an electrical output was directly available which was proportional to the input square, e.g. $e^2(t)$. To determine the mean power spectrum, the recorder output of a Hewlett–Packard 302A wave analyser was passed through the heater of an insulated vacuum thermocouple to produce a thermocouple voltage proportional to the instantaneous power spectrum of the input $e^2(f,t)$ which could then be preamplified and averaged with the analogue circuit to give the mean power spectrum $\phi_\theta(f) = \overline{e^2(f,t)}$. The exact frequency tuned by the heterodyne wave analyser was measured with a Berkeley Universal Counter-Timer. The analogue averager had a variable time-constant which could be adjusted between 2 sec and 30 min. A statistical analysis of the averaging operations is given by Priestley & Gibson (1962).

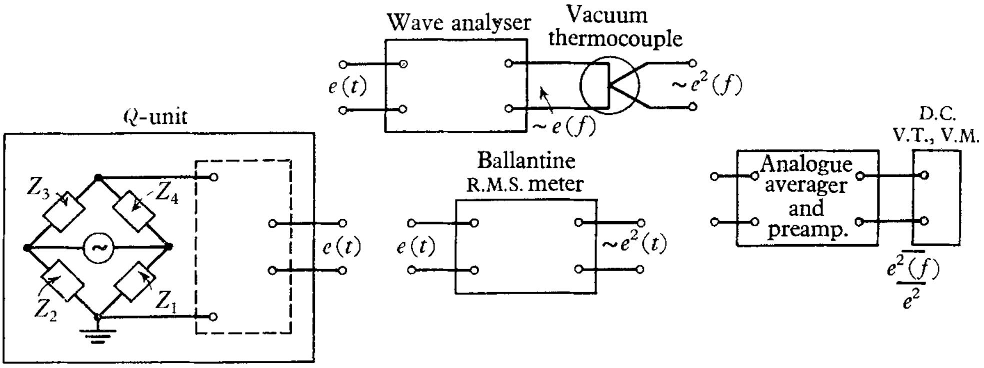


FIGURE 4. A.C. Wheatstone Bridge circuit (Q-unit).

5. Conclusions

Using the single-electrode probe in the circuit of figure 3, noise levels equivalent to only 0·003 % R.M.S. conductivity fluctuations were possible. Thus, concentration fluctuations of $(\overline{\Delta C^2})^{\frac{1}{2}}/\bar{C} = 0{\cdot}003\,\%$ or temperature fluctuations of $(\overline{\Delta T^2})^{\frac{1}{2}} = 0{\cdot}001$ °C could be measured at a signal-to-noise ratio of one. The equivalent noise level reported by Manning (1959, p. 37) was $(\overline{\Delta C^2})^{\frac{1}{2}}/\bar{C} = 0{\cdot}4\,\%$, which shows that by employing the inherent advantages of bridge circuitry, signals of energy four orders of magnitude smaller could be detected.

The spatial resolution of the conductivity probe was estimated using a mathematical model which indicated that the signal was attenuated about 10 % for spatial fluctuations of $100r_0$. For a 0·0004 in. probe, this corresponds to a wave-number of about 126 cm^{-1}. For the laboratory experiments discussed by Gibson & Schwarz (1963), this wave-number is near the region of the diffusive cut-off. Also, by comparing the scalar dissipation measured by $6D_\theta \int k_1^2 \phi_\theta(k_1)\, dk_1$ with that calculated from the decay of $\overline{\theta^2}$-stuff behind a grid, it was found that the probe was attenuating the very high wave-number signals at a somewhat greater rate than predicted. This has been interpreted as an indication that it was the hydrodynamic interference of the probe tip which controlled the spatial resolution of this probe, rather than electrode size.

This work has been supported by the National Science Foundation under grants G 5506 and G 12141.

REFERENCES

Cairns, E. J. & Prausnitz, J. M. 1960*a* *A.I.Ch.E. Journal* **6**, 400.

Cairns, E. J. & Prausnitz, J. M. 1960*b* *A.I.Ch.E. Journal* **6**, 554.

Gibson, C. H. & Schwarz, W. H. 1963 *J. Fluid Mech.* **16**, 365.

Glasstone, S. 1942 *An Introduction to Electrochemistry*. New York: D. Van Nostrand Co., Inc.

Jones, Grinnell *et al.* *J.A.Ch. Soc.* **50**, 1049 (1928); **51**, 2407 (1929); **53**, 411 (1931); **53**, 1207 (1931); **55**, 1780 (1933); **57**, 272 (1935); **57**, 280 (1935); **59**, 731 (1937).

Lamb, D. E., Manning, F. S. & Wilhelm, R. H. 1960 *A.I.Ch.E. Journal* **6**, 682.

Manning, F. S. 1959 Ph.D. Thesis (Chemical Engineering) Princeton University.

Prausnitz, J. M. & Wilhelm, R. H. 1956 *Rev. Sci. Instrum.* **27**, 941.

Priestley, M. & Gibson, C. H. 1962 Estimation of power spectra by a Wave Analyzer. *Stanford University Appl. Math. and Stat. Lab. Tech. Rep.* no. 5.